AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

A-level MATHEMATICS

Unit Statistics 3

Monday 25 June 2018 Morning Time allowed: 1 hour 30 minutes

Materials
For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
TOTAL	

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

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Answer **all** questions.

Answer each question in the space provided for that question.

1 Alfie, Bishen and Chao are three male students. They selected, each from a different population, a random sample of 12 pairs of measurements.

Each student then calculated the value of the product moment correlation coefficient, r, based on his pairs of measurements.

The values obtained are shown in the table.

Student	**Value of r obtained**
Alfie	-0.732
Bishen	0.556
Chao	1.207

Each student intended to use his value of r to test the null hypothesis $\mathrm{H}_0 : \rho = 0$ against the alternative hypothesis $\mathrm{H}_1 : \rho > 0$, where ρ denotes the product moment correlation coefficient for the population from which his sample was selected.

(a) Name the two students for whom it is unnecessary to carry out the test. Give a justification for each student that you select.

[2 marks]

(b) Carry out the test, using the 5% level of significance, for the other student.

[3 marks]

QUESTION PART REFERENCE

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2 The internal telephone calls received by a hotel's reception desk, during the 13 hours between 8.00 am and 9.00 pm on weekdays, can be assumed to occur randomly and independently, and at a constant average rate of λ **per hour**.

During a period of five weekdays, a total of 1521 such calls was received.

Construct an approximate 99% confidence interval for λ.

[6 marks]

QUESTION PART REFERENCE

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3 Manufacturer *CoMB* has three factories, A, B and C, which produce computer motherboards.

Factory A produces 25%, factory B produces 35% and factory C produces 40% of *CoMB*'s motherboards.

The **percentage** of non-conforming motherboards produced by each factory is shown in the table.

Factory	Percentage of non-conforming motherboards
A	1.5
B	1.0
C	0.5

(a) Calculate the probability that a randomly selected motherboard produced by *CoMB*:

(i) is non-conforming;

[2 marks]

(ii) was produced at factory C, given that it is non-conforming.

[2 marks]

(b) Calculate the probability that in three randomly selected non-conforming motherboards produced by *CoMB*, one was produced at factory A, one was produced at factory B and one was produced at factory C.

[3 marks]

(c) *CoMB* closes factory A and moves its production of motherboards to factory C. This has no effect on the percentages of non-conforming motherboards produced by factories B and C.

Calculate the probability that, following this change, a randomly selected motherboard produced by *CoMB* was produced at factory C, given that it is non-conforming.

[3 marks]

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4 The durations of telephone calls to a company's UK helpline can be modelled by a normal distribution with mean μ_{UK} and standard deviation 3.2 minutes.

The durations of telephone calls to the same company's overseas helpline can be modelled by a normal distribution with mean μ_{OS} and standard deviation 4.7 minutes.

A random sample of 25 telephone calls to the UK helpline had a mean duration of 10.8 minutes.

An independent random sample of 16 telephone calls to the overseas helpline gave the following durations in minutes.

13.7	16.2	15.4	11.7	18.1	18.0	9.4	12.6
17.8	9.2	13.6	23.4	16.3	14.9	25.1	12.6

(a) Construct a 98% confidence interval for $\mu_{OS} - \mu_{UK}$.

[5 marks]

(b) What does your confidence interval reveal? Justify your answer.

[2 marks]

(c) The company's management team subsequently decides to increase the sample sizes of 25 and 16 to the same value, n, so that a resultant 98% confidence interval for $\mu_{OS} - \mu_{UK}$ would have a width of at most 2 minutes.

Calculate, to the nearest 5, the minimum value of n.

[4 marks]

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5 Lorna, a statistician, is also a keen gardener. Over many years, she has established the following summary information on the time, in minutes, for each of the tasks that she carries out to maintain her property's lawns.

Task	Variable	Mean	Variance
Get out equipment	U	5	1
Mow grass	V	30	25
Trim edges	W	15	9
Sweep up and clean equipment	X	25	16
Put away equipment	Y	10	4

(a) (i) The random variables V and W are such that $\rho_{VW} = 0.80$.

Determine values for the mean and the variance of $M = V + W$.

[3 marks]

(ii) The random variables M, U, X and Y are independent.

Determine values for the mean and the variance of $D = M - (U + X + Y)$.

[2 marks]

(b) Assuming that each of the variables M and D is approximately normally distributed, determine the probability that Lorna takes:

(i) more than one hour to mow the grass and trim the edges of her property's lawns;

(ii) less time to mow the grass and trim the edges of her property's lawns than to carry out the other lawn maintenance tasks.

[5 marks]

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6 (a) The random variable X has the distribution $\mathrm{B}(n, p)$ for which $\mathrm{E}(X) = np$.

(i) Prove that $\mathrm{Var}(X) = np(1-p)$.

[5 marks]

(ii) Show that $\mathrm{P}(X = x) = \left(\frac{n-x+1}{x}\right)\left(\frac{p}{1-p}\right)\mathrm{P}(X = x-1)$.

[2 marks]

(b) The random variable Y has a binomial distribution with $\mathrm{E}(Y) = 7.5$ and $\mathrm{P}(Y = 8) = \mathrm{P}(Y = 7)$.

Find the value of $\mathrm{Var}(Y)$.

[3 marks]

(c) The random variable U has a binomial distribution with $\mathrm{E}(U) = 5$ and $\mathrm{Var}(U) = 4.995$.

Use a distributional approximation to estimate $\mathrm{P}(U \leqslant 3)$. Justify your choice of distributional approximation.

[3 marks]

(d) The random variable V has a binomial distribution with $\mathrm{E}(V) = 60.5$ and $\mathrm{P}(V = 61) = \mathrm{P}(V = 60)$.

Use a distributional approximation to estimate $\mathrm{P}(V > 50)$.

[3 marks]

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7 *Delquick* delivers parcels for an online retailer to private households.

Delquick claims that more than 25 per cent of such parcels are delivered early, before the estimated delivery date stated by the retailer.

(a) To investigate this claim, the retailer monitored the delivery of a random sample of 30 parcels delivered by *Delquick*, and found that 10 were delivered early.

Use an exact test to show that *Delquick*'s claim is not supported at the 5% level of significance by these data.

[6 marks]

(b) (i) *Delquick* subsequently employed an independent agency to investigate the claim. The agency selected a random sample of 300 parcel deliveries and found that 90 were delivered early.

Use an approximation to the distribution of the **sample proportion**, $\widehat{P}$, and the 5% level of significance to investigate *Delquick*'s claim that more than 25 per cent of parcels are delivered early.

[4 marks]

(ii) Show that the critical value of $\widehat{P}$ for the test in part **(b)(i)** is 0.291, correct to three significant figures.

[2 marks]

(iii) Assuming that 30 per cent of parcels are actually delivered early, estimate the power for a test of the claim that more than 25 per cent of parcels are delivered early, based on a random sample of 300 parcel deliveries and using the 5% level of significance.

[5 marks]

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